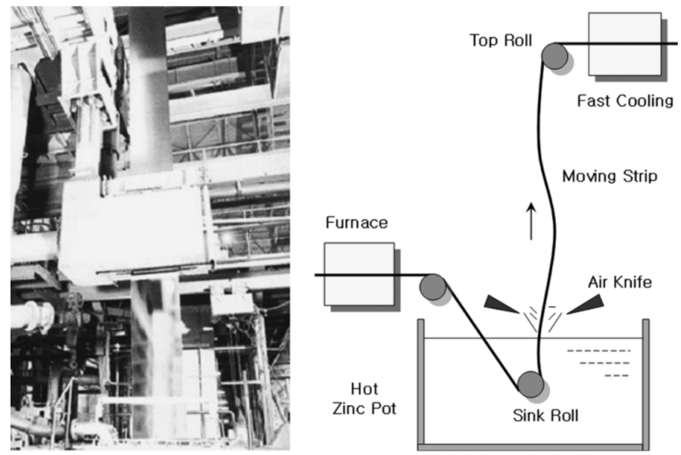
# Energy-Based Control of Axially Translating Beams: Varying Tension, Varying Speed, and Disturbance Adaptation

Kyung-Jinn Yang, Keum-Shik Hong, and Fumitoshi Matsuno

***Abstract*—In this brief, the investigational results for a robust adaptive vibration control of a translating tensioned beam with a varying traveling speed are presented. The dynamics of beam and actuator is modeled via the extended Hamilton's principle, in which the tension applied to the beam is given as a nonlinear spatiotemporally varying function. The moving beam is divided into two parts, a controlled span and an uncontrolled span, by a hydraulic touch-roll actuator that is located in the middle section of the beam. The transverse vibration of the controlled span is suppressed by the touch-roll actuator, whereas the vibration of the uncontrolled span is treated as a disturbance, and the magnitude of unknown disturbance is estimated. In a proper mathematical manner, the Lyapunov method is employed to design robust adaptive boundary control laws for ensuring the vibration reduction of the nonlinear time-varying system, and also to ensure the stability of the closed-loop system. The effectiveness of the proposed controller is demonstrated via numerical simulations.**

***Index Terms*—Axially moving continua, Lyapunov method, robust adaptive control, stability, uniform ultimate boundedness.**




(a) (b)

Fig. 1. Vertically moving steel strip in the zinc galvanizing line. (a) Picture (POSCO, Korea). (b) Schematic.

## I. Introduction

THE control problem of axially moving continua occurs in such high-performance mechanical systems as cranes, strips in a thin metal-sheet production line, high-rise elevators, chains and belts, high-speed magnetic tapes, paper sheets under processing, and deployable robot arms as well. However, unwanted vibrations of moving continua due to the flexibility property and time-varying conditions restrict the utility of the systems in many applications, and in particular in high-speed, precision systems.

For an example, Fig. 1 shows a continuous hot-dip zinc galvanizing process. The steel strips, of order of 1∼1.2 m wide by 0.8∼3.0-mm thick, are preheated and passed at a constant speed through a pot of molten zinc at a temperature in the region of 450 °C. A zinc film is entrained onto the strip as it emerges from the pot. In order to achieve the target deposited mass and maintain it over various process conditions, a pair of air knives, which direct a long thin wedge-shaped jet of high-velocity air onto the strip, are generally used to control the deposited mass by stripping out excess zinc back into the pot. The deposited film solidifies while the strip moves vertically upward, cooling as it goes, and horizontally for about 40 m, to the gauge which measures the mass of zinc deposited on the strip surfaces. Here, two control objectives for the galvanizing line are to improve the uniformity of the zinc deposit on the strip surfaces and to reduce the zinc consumption. The transverse movement and vibration of the strip is known to be the main cause of the difference between the average deposited masses on the right and left surfaces and the nonuniformity across the surfaces. A regulation problem of deposited mass by adjusting the gap between the strip and the air knives has been studied [1]. However, a pertinent problem there was the lack of a precise knowledge of strip position due to the vibration of the strip. Many galvanized steel manufacturers including POSCO (Korea) and U.S. Steel have attempted to measure the strip position directly by installing laser transducers near the air knives. However, no long-term success has been reported yet, because the high-temperature environment makes the transducers unreliable. Thus, as a means of avoiding use of unreliable transducers, the strip vibrations need to be directly suppressed by using a more practical and reasonable method such as active boundary force control of axially moving continua.

Vibration control schemes for axially moving strings include [2]–[6] and others. Those for axially moving beams include [7]–[11], among others. Robust and adaptive control schemes for hyperbolic distributed systems include [12]–[14]. It is notable that most studies were limited either to cases with constant spatial tension and transport velocity or to nonaxially moving (stationary) systems. However, in practical situations, the effects of velocity changes might be significant, and so almost all

Manuscript received June 20, 2003; revised September 13, 2004. Manuscript received in final form April 8, 2005. Recommended by Associate Editor G. Rosen. The work of K.-J. Yang was supported by the Postdoctoral Fellowship Program of the JSPS, Japan. The work of K.-S. Hong was supported by the Research Center for Logistics Information Technology (LIT) designated by the Ministry of Education and Human Resources Development of Korea.

K.-J. Yang and F. Matsuno are with the Department of Mechanical Engineering and Intelligent Systems, University of Electro-Communications, Tokyo 182-8585, Japan (e-mail: jinnky@hi.mce.uec.ac.jp; matsuno@hi.mce.uec.ac.jp).

K.-S. Hong is with the School of Mechanical Engineering, Pusan National University, Busan 609-735, Korea (e-mail: kshong@pusan.ac.kr).

Digital Object Identifier 10.1109/TCST.2005.854368

axially moving systems have a varying tension that is a function of both time and space due to longitudinal accelerations and gravity, and/or the eccentricity of a support roller, and/or external disturbances, among other factors [3], [6], [11], [14]. Also, the terminology, boundary control, may not make sense for axially moving continua when the control input is applied at an inner point of the span, because the span of interest and the adjacent span are connected and the vibrations of the adjacent span would occur continuously regardless of the exerted control input [3], [6], [11]. This means, when considering axially moving continua affected by the adjacent span, how to handle the effect of the vibrations from the adjacent span also becomes a very important point in order to obtain an effective controller for ensuring vibration reduction. Hence, to achieve better control performance, a novel active controller incorporating spatiotemporally varying tension, time-varying traveling velocity, and the vibration effect from the adjacent span should be investigated.

In this brief, a nonlinear tensioned beam translating at a time-varying speed is focused on, resulting in a problem formulation, an implementable controller design, and a stability analysis, under the assumption that the motion of the adjacent span can be treated as a disturbance to the span of interest. Fig. 2 shows a schematic of the control strategy of an axially moving beam using a hydraulic touch-roll actuator. The axially moving beam is divided into two spans, that is, a controlled span and an uncontrolled span, by a transverse force actuator, as shown in Fig. 2. The main objective is to suppress the transverse vibration in the controlled span despite the unknown undesired effect arising from the uncontrolled span. By employing the extended Hamilton's principle, dynamics of beam and actuator are modeled, in which the tension applied to the beam is given as a nonlinear spatiotemporally varying function due to the traveling speed variation. Through the dynamic models and the Lyapunov energy method, a robust adaptive controller effective for vibration reduction was designed. Since the proposed control laws depend on the displacement and slope measurements on the controlled span side of the actuator, the vibration suppression of the axially moving beam can be successfully implemented. Further, the proposed control scheme can be directly applied to the axially moving string system.

This brief is structured as follows. In Section II, time-varying beam equations of motion and their boundary conditions are derived, and the problems are formulated. In Section III, robust adaptive boundary control laws to suppress the transverse vibrations of the beam are derived, and the stability of the closed-loop system is investigated. In Section IV, simulation results are demonstrated. Conclusions are given in Section V.

## II. Beam Model: Problem Formulation

In Fig. 2, the boundary rolls at $x = 0$ and $x = l_T$ are assumed to be, that is, fixed in the sense that there is no vertical movement, but this allows the beam to move in a horizontal direction. The two touch rolls, where the control input (force) is exerted, are located at $x = l$ in the middle section of the beam. Note that the vibrational energy of the uncontrolled span could not converge to zero despite the control actuator at $x = l$, since the

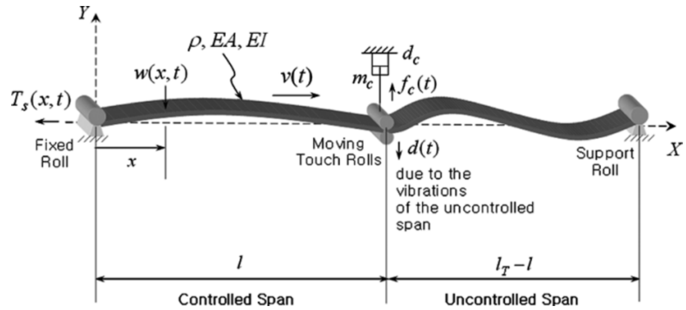



Fig. 2. Schematic of a translating beam subject to an active vibration control.

input control law is generally designed only by considering the controlled span part. Hence, uniform ultimate boundedness can be concluded in this case, which will be proved in the sequel. Such undesired vibration of the uncontrolled span gives the effect to the hydraulic actuator of an unexpected external force. Hence, applying a controller designed under ignorance of the vibrational effect from the adjacent span might bring about an unforeseen result.

In this brief, the unexpected external force applied to the actuator is treated as an unknown right boundary disturbance on the controlled span part, since the touch rolls at $x = l$ function as the right boundary of the controlled span. Such a strategy allows us to consider only the controlled span part of the beam regardless of the adjacent span, in designing an effective boundary control law; then the burden of complicated system analysis when considering entire spans can be avoided.

From Fig. 2, let $t$ be the time, $x$ be the spatial coordinate along the longitude of motion, $v(t)$ be the varying axial speed of the beam, $v(t) > 0$ for all $t$, $w(x,t)$ be the transversal displacement of the beam at spatial coordinate $x$ and time $t$, and $l$ be the length of the controlled span. Also, let $\rho$ be the mass per unit length, $A$ be the cross-sectional area, $E$ be the coefficient of elasticity, $I$ be the moment of inertia of the beam cross section, and $T_s(x,t)$ be the spatiotemporally varying tension applied to the beam. Let the mass and damping coefficients of the hydraulic actuator be $m_c$ and $d_c$, respectively. The control force $f_c(t)$ is applied to the touch rolls to suppress the transverse vibrations of the axially moving beam, and $d(t)$ denotes the unknown disturbance force exerted on the actuator due to the transverse vibration of the uncontrolled span.

The translating continua system has to be analyzed in view of Eulerian description, since our attention is focused on what happens on the moving continua in the specific region of interest as time passes [15], [16]. Hence, the kinetic energy in the transverse direction of the beam should be given with the total derivative operator (material derivative) with respect to time, due to the axial speed $v(t)$. That is, the kinetic and potential energies of the axially moving beam between $x = 0$ and $x = l$ including the hydraulic actuator are given as, respectively

$$\begin{aligned} T &= T_{\text{beam}} + T_{\text{act}} \\ &= \frac{\rho A}{2} \int_0^l (w_t + v w_x)^2 dx + \frac{1}{2} m_c w_t^2(l) \end{aligned} \tag{1}$$

$$U_{\text{beam}} = \int_0^l \left\{ \frac{T_s(x,t)}{2} w_x^2 + \frac{EA}{8} w_x^4 + \frac{EI}{2} w_{xx}^2 \right\} dx \quad (2)$$

where $w(l) = w(l,t)$ for notational brevity. The first term on the right side in (2) is caused by the beam tension, the second term reflects the strain energy due to axial stiffness, and the last term comes from the bending moment [9].

The equations of motion and the boundary conditions can be obtained through Hamilton's principle. However, in translating systems, the configurations at the end times of the variational principle are not prescribed [15], [17]. Hence, new approaches for d'Alembert's principle are required, which can be accomplished by introducing a general theory for calculating the time rate of change [15], [18]. That is, by applying the general theory to the variational principle, the property in the system volume is converted to that in the control volume. Since the configurations in the control volume are prescribed at specific times in Eulerian description, a novel extended Hamilton's principle for translating continua systems can be established without loss of the generality of the classic Hamilton's principle, and which is given as [4], [15]

$$\delta \int_{t_0}^{t_f} L dt + \int_{t_0}^{t_f} \delta W_{n.c.} dt - \int_{t_0}^{t_f} \rho A \left( w_t(l) + v w_x(l) \right) \cdot v \delta w(l) dt = 0 \quad (3)$$

where $L = T - U_{\text{beam}}$ and $\delta W_{n.c.} = (f_c - d_c w_t(l) + d(t)) \delta w(l)$.

From (3), the equations of motion and the boundary conditions of the axially moving beam system of the controlled span part in Fig. 2 are finally derived as

$$\rho A (w_{tt} + \dot{v} w_x + 2 v w_{xt} + v^2 w_{xx}) - \left( (T_s w_x)_x + \frac{3EA}{2} w_x^2 w_{xx} \right) + EI w_{xxxx} = 0 \quad (4)$$

$$w(0) = 0, \quad w_x(0) = 0, \quad w_{xx}(l) = 0 \quad (5)$$

$$m_c w_{tt}(l) + d_c w_t(l) + T_s(l) w_x(l) + \frac{EA}{2} w_x^3(l) - EI w_{xxx}(l) - d(t) = f_c(t). \quad (6)$$

Note that (4) is a nonlinear hyperbolic PDE representing the transverse motion of the moving beam, whereas (6) is an ODE describing the motion of the hydraulic actuator in compliance with the transversal control force at $x = l$. The term $(T_s + 3EA w_x^2/2)$ in (4) is often called a nonlinear tension [9]. The moving speed $v$, to avoid a divergence of the solution, should be smaller than the critical speed [9], [10]. Following [8], the tension $T_s(x,t)$ in (4) is given as:

$$T_s(x,t) = T_0 - \rho A (l - x)(eg - \dot{v}) \quad (7)$$

where $e = 0$ for the horizontally translating beam, $e = 1$ for the vertically translating beam, and $g$ and $T_0$ denote the gravitational acceleration and the initial tension applied to the beam, respectively. Note that the axial force may become a tensile and compressive force during the deceleration ($\dot{v} < 0$) and acceleration ($\dot{v} > 0$), respectively, of the beam.

Since the tension $T_s(x,t)$ is a spatiotemporally varying function, the tension variation has to be incorporated into the control law design. Provided that there is no large external disturbance to the system, $T_s(x,t)$ can be assumed to be continuous and uniformly bounded, $0 < T_{s,\min} \le T_s(x,t) \le T_{s,\max}$, $|(T_s)_t| \le (T_s)_{t,\max}$, and $|(T_s)_x| \le (T_s)_{x,\max}$ for all $x \in [0,l]$, $t \ge 0$, and some *a priori* known constants $T_{s,\min}$, $T_{s,\max}$, $(T_s)_{t,\max}$, and $(T_s)_{x,\max}$, where $(T_s)_t = \rho A (l - x) \ddot{v}$ and $(T_s)_x = \rho A (eg - \dot{v})$ from (7). Considering practical situations such as a high-tensioned beam under axial transport processing, it can be assumed that the lower bound $T_{s,\min}$ is larger than both $(T_s)_{t,\max}$ and $(T_s)_{x,\max}$ due to the high tension limit. However, for some visco-elastic materials such as synthetic rubber and synthetic fiber, in which such a high tension is not required, the fluctuating $v(t)$ might not guarantee $T_{s,\min} > \max[(T_s)_{t,\max}, (T_s)_{x,\max}]$.

Now, consider the open-loop controlled beam system in (4)–(6) with the assumption of $f_c = d = 0$. From (1) and (2), the total vibrational energy $E(t)$ of the beam system is given by

$$E(t) = (T_{\text{beam}} + U_{\text{beam}}) + T_{\text{act}} = E_{\text{beam}} + T_{\text{act}}. \quad (8)$$

Applying the general theory for calculating the time rate of change in [15] and [18] to $E_{\text{beam}}(t)$ in (8) yields

$$\dot{E}_{\text{beam}}(t) = \int_0^l \frac{\partial}{\partial t} \widetilde{E}_{\text{beam}}(x,t) dx + v \widetilde{E}_{\text{beam}}(x,t) \Big|_0^l \quad (9)$$

where $2\widetilde{E}_{\text{beam}}(x,t) = [\rho A (w_t + v w_x)^2 + T_s w_x^2 + (EA/4) w_x^4 + EI w_{xx}^2]$. That is, because the system involves a mass flow entering in and out at the boundaries, the net change of the total energy $E_{\text{beam}}(t)$ is the sum of the change in the control volume and the energy flux at the boundaries.

Hence, the time derivation of $E(t)$ in (8) is evaluated as

$$\begin{aligned}\dot{E}(t) = &- EI v w_{xx}^2(0) - d_c w_t^2(l) \\ &+ v w_x^2(l) \left\{ T_s(l) + \frac{EA}{2} w_x^2(l) \right\} - v w_x(l) EI w_{xxx}(l) \\ &+ \frac{1}{2} \int_0^l \left\{ (T_s)_t + v (T_s)_x \right\} w_x^2 dx. \end{aligned} \quad (10)$$

From (10), it is justified that, when the time rate of change of $T_s(x,t)$ is a positive value, it increases the mechanical energy to a factor of $w_x^2$. Hence, it should be properly handled in order to decrease the vibration energy of the beam. And, it is also seen that the open-loop system controlled only by a damper at the right boundary is not effective to suppress the vibrations, since the stability of the open-loop system is uncertain if determined from (10), excepting the slowly-moving or stationary continua system (i.e., $v = 0$). Further, in the process of derivation, the effect of the boundary disturbance from the uncontrolled span is disregarded. Thus, to surmount the time-varying property and the boundary disturbance as well, a novel control scheme is required.

## III. Control Law

The considered translating beam in (4)–(6) takes the form of a distributed parameter system, not to mention the form of a nonlinear time-varying system. The Lyapunov method can cope with the time-varying nature of the system, and also the resulting control law and stability arguments can be applied rigorously to the distributed parameter system since introducing the spatial approximations can be avoided in the application of Lyapunov concepts. Due to these reasons, the Lyapunov method is employed to design an effective and implementable robust adaptive boundary controller for axially moving continua systems.

As shown in (4)–(6), the control mechanism is coupled with the beam system because the controller is attached to the right boundary of the controlled beam, on which the control force $f_c$ is applied. To obtain the stability of coupled system (4)–(6), the convergence of the boundary actuator should also be satisfied. Hence, a modification of the total mechanical energy in (8) is necessary in order to obtain an appropriate Lyapunov function candidate for the coupled system.

The beam vibration energy $E_{\text{beam}}$ in (8) and the following function are equivalent [3]:

$$V_{\text{beam}} = E_{\text{beam}} + \beta\rho A \int_0^l x w_x (w_t + v w_x) dx \tag{11}$$

where $0 < \beta < \min\{l^{-1}v, \beta_1^{-1}\}$ and $\beta_1 = l \cdot \max\{1, \rho A T_{s,\min}^{-1}\}$. Note that, according to (5) and Poincare's inequality, the stability of the hydraulic actuator system can be analyzed by adding the slope term at $x = l$ to the mechanical energy [2]. Also, since the boundary disturbance is really bounded under the condition of the bounded energy of the uncontrolled span (see Remark 3 following), the boundary disturbance $d(t)$ is assumed to be uniformly bounded by $\mu_d$, that is., $\mu_d \geq |d(t)|$ for all $t$, where $\mu_d$ is an unknown positive constant.

Thus, a positive–definite functional $V(t)$, as the total energy of the moving beam system including the actuator, is defined as

$$V(t) = V_{\text{beam}} + V_{\text{act}} + V_{dist} \tag{12}$$

where $V_{\text{act}} = m_c\{w_t(l) + (v+\beta l) w_x(l)\}^2/2$, $V_{dist} = \widetilde{\mu}^2/2\gamma_d$ and $\widetilde{\mu}_d = \hat{\mu}_d - \mu_d$, and where $\hat{\mu}_d$ is the adaptive estimate of $\mu_d$, which will be specified in the sequel. In the work, the functional $V(t)$ in (12) is considered as a Lyapunov function candidate in order to show the stabilization of the closed-loop system by applying a robust adaptive boundary controller, which will be designed. The last term in (12) is added as a pseudo-energy to ensure that the desired final state, $(w, \dot{w}, w(l), w_t(l), \tilde{\mu}_d) = (0,0,0,0,0)|_{\text{desired}}$, is the unique minimum of $V(t)$ in (12). From (5) and Poincare's inequality, it is straightforward to confirm that the positive system parameters guarantee $V \geq 0$ and that indeed the global minimum of $V = 0$ is attained only in the desired state.

Now, the robust control law for the right boundary control force $f_c(t)$ is proposed as

$$f_c(t) = -m_c\{\dot{v} w_x(l) + (v+\beta l) w_{xt}(l)\} + d_c w_t(l) - \frac{\beta\rho A l v}{v+\beta l} w_t(l) + f_d(t) \tag{13}$$

where the additional term $f_d(t)$ is regarded as a new input signal determined as based on robust control strategy [12] and is given by

$$f_d(t) = -\frac{\hat{\mu}_d^2(t)}{\hat{\mu}_d(t)\,|\overline{w}(l)| + \varepsilon_d}\overline{w}(l) \tag{14}$$

where $\bar{w}(l) = \{w_t(l) + (v+\beta l) w_x(l)\}$ and $\varepsilon_d > 0$. The adaptation law $\hat{\mu}_d$ in (14) is proposed as

$$\dot{\hat{\mu}}_d(t) = -\delta_d \hat{\mu}_d(t) + \gamma_d\,|\overline{w}(l)| \tag{15}$$

where $\delta_d > 0$ and $\gamma_d > 0$.

Let $\hat{\mathbf{T}}_{\mathbf{s}} \equiv \beta(T_{s,\min} - \rho A v^2) - (T_s)_{t,\max} - (\beta l + v)(T_s)_{x,\max}$ and $T_s(l) > \rho A v^2$. By employing the derivation method used in (9) and also by substituting the robust adaptive control laws (13)–(15) into (6), the time rate of change of the energy functional $V(t)$ in (12) is evaluated as

$$\begin{aligned} \dot{V}(t) \leq & -EIvw_{xx}^2(0) - \frac{\beta\rho A}{2}\int_0^l w_t^2 dx \\ & - \frac{1}{2}\hat{\mathbf{T}}_{\mathbf{s}}\int_0^l w_x^2 dx - \frac{3\beta EI}{2}\int_0^l w_{xx}^2 dx \\ & - \frac{\beta l}{2}\left(T_s(l) - \rho A v^2\right) w_x^2(l) \\ & - \beta\rho A l\left(\frac{v}{v+\beta l} - \frac{1}{2}\right) w_t^2(l) + \theta(t) \end{aligned} \tag{16}$$

where $\theta(t) \equiv -(\delta_d/\gamma_d)\widetilde{\mu}_d^2 + \varepsilon_d + (\delta_d/2\gamma_d)\mu_d$. To derive the last term in (16), the following relationship is utilized [12]:

$$\begin{aligned} \{w_t(l) + (v+\beta l) w_x(l)\}(f_d + d) + \frac{1}{\gamma_d}\widetilde{\mu}_d\dot{\hat{\mu}}_d \\ \leq -\frac{\delta_d}{\gamma_d}\widetilde{\mu}_d^2 + \varepsilon_d + \frac{\delta_d}{2\gamma_d}\mu_d. \end{aligned}$$

From $\beta < \min\{l^{-1}v, \beta_1^{-1}\}$ in (11), the second term at the end in (16) is negative, that is, $v/(v+\beta l) > 0.5$, and so (16) becomes

$$\begin{aligned} \dot{V}(t) \leq & -\min\left[3\beta, \frac{1}{2T_{s,\max}}\hat{\mathbf{T}}_{\mathbf{s}}, \frac{\beta}{2}, \frac{1}{4\rho A v^2}\hat{\mathbf{T}}_{\mathbf{s}}\right] \\ & \times \frac{1}{1+\beta\beta_1} V_{\text{beam}} \\ & - \min\left\{\frac{\beta l\left(T_s(l) - \rho A v^2\right)}{2m_c(v+\beta l)^2}, \frac{\beta\rho A l}{m_c}\left[\frac{v}{v+\beta l} - \frac{1}{2}\right]\right\} \\ & \times V_{\text{act}} + \theta(t). \end{aligned} \tag{17}$$

Finally, the main theorem of this brief is established.

*Theorem 1:* Suppose $\{\beta(T_{s,\min} - \rho A v^2) - (T_s)_{t,\max} - (\beta l + v)(T_s)_{x,\max}\} > 0$ and $T_s(l) - \rho A v^2 > 0$. Then, the dynamics of the closed-loop system in (4)–(6) controlled by the robust adaptive control laws in (13)–(15) is uniformly ultimately bounded, that is

$$\dot{V}(t) \leq -\lambda \times (V_{\text{beam}} + V_{\text{act}}) + \theta(t) \tag{18}$$

where $\lambda > 0$ and

$$\lambda = \min\left[\frac{1}{(1+\beta\beta_1)}\left\{3\beta, \frac{\hat{\mathbf{T}}_{\mathbf{s}}}{2T_{s,\max}}, \frac{\beta}{2}, \frac{\hat{\mathbf{T}}_{\mathbf{s}}}{4\rho A v^2}\right\} \frac{\beta l\left(T_s(l) - \rho A v^2\right)}{2m_c(v+\beta l)^2}, \frac{\beta\rho A l}{m_c}\left(\frac{v}{v+\beta l} - \frac{1}{2}\right)\right].$$

Since $\theta(t)$ in (18) is uniformly bounded as $-(\delta_d/\gamma_d)\widetilde{\mu}_d^2 + \varepsilon_d + (\delta_d/2\gamma_d)\mu_d$, the uniform ultimate boundedness region of $V(t)$

can be made arbitrarily small by a suitable choice of $\varepsilon_d$, $\delta_d$, and $\gamma_d$.

From Theorem 1, it is concluded that the dynamics of the closed-loop system is uniformly ultimately bounded. Also, note that $\theta(t)$ in (18) can be pushed to an arbitrarily small boundedness region by making $\varepsilon_d$, $\delta_d$ sufficiently small and $\gamma_d$ relatively large. As the result, the uniform ultimate boundedness region of $V(t)$ can be made arbitrarily small [19], which implies that all state variables of the closed-loop system decay to near zero in time.

*Remark 1:* From Theorem 1, it is seen that, if the effect of the disturbance $d(t)$ from the uncontrolled span is ignored, that is, $V_{dist} = 0$ in (12), then the boundary controller $f_c$ alone can make the closed-loop system exponentially stable, even without employing the robust control term $f_d$ in (13). That is, $V(t) \leq V(0)\, e^{-\lambda t}$ from (18) due to $\theta(t) = 0$. Also, in that case, and remembering the discussion in Section II, the open-loop system with only the passive damper can be stable if the damping coefficient is sufficiently large. However, the damping value never gives any effect to the slope term, as shown in (10), and this means that however large the damping value, the performance of the open-loop system cannot be improved [7]. But the point of this study is only to show how to handle the effect of the disturbance $d(t)$, since it cannot simply be neglected in actual systems. Hence, considering the effect of $d(t)$, the closed-loop system controlled by only $f_c$ without $f_d$ cannot guarantee stability, not to mention the exponential stability.

As mentioned in Remark 1, no stability conclusion can be drawn from the Lyapunov function candidate $V(t)$ in (12), since the time derivation of $V(t)$ in (18) may take positive values because of the last term $\theta(t)$, which implies that the disturbance $d(t)$ due to the transverse vibration of the uncontrolled part causes an increase in the mechanical energy of the controlled span part of the beam. Nevertheless, by Theorem 1, it has been investigated that the proposed robust adaptive boundary controller proposed assures the boundedness of all signals in the closed-loop system and the convergence near to zero. However, it should be noted that Theorem 1 is a sufficient condition but not a necessary condition since the value of $(T_s)_t$ is impossible to maintain as a positive value for all $t$ in an actual system, given that it is close to a periodic pattern. Hence, Theorem 1 does not mean that any system dominated by a varying tension with unknown frequency pattern satisfying $\beta T_{s,\min} < (T_s)_{t,\max}$ is always diverged. However, in actual processing lines, almost all translating continua operate under a high-tensioned condition, and so the proper control gains satisfying the conditions in Theorem 1 can be assured.

*Remark 2:* Robust adaptive control laws (13)–(15) are given for velocity $w_t(l)$, slope $w_x(l)$, and slope rate $w_{xt}(l)$ at $x = l$, not using the system parameters $T(l)$ and $EI$. By using an encoder (or photodiode) on the actuator and two laser sensors, the actuator displacement $w(l)$ and the slope $w_x(l)$ on the actuator, respectively, can be measured [6], [11]. The actuator velocity $w_t(l)$ and the slope rate $w_{xt}(l)$ can then be implemented by the backward differencing of the signals. Here, an important point to be noted is that the slope $w_x(l)$, as the control input signal, should be measured on the controlled span side of the actuator at $x = l$, not on the uncontrolled span side. If the boundary slope is measured on the uncontrolled span side of the actuator, the closed-loop system in (4)–(6) can be unstable, which will be explained in the sequel.

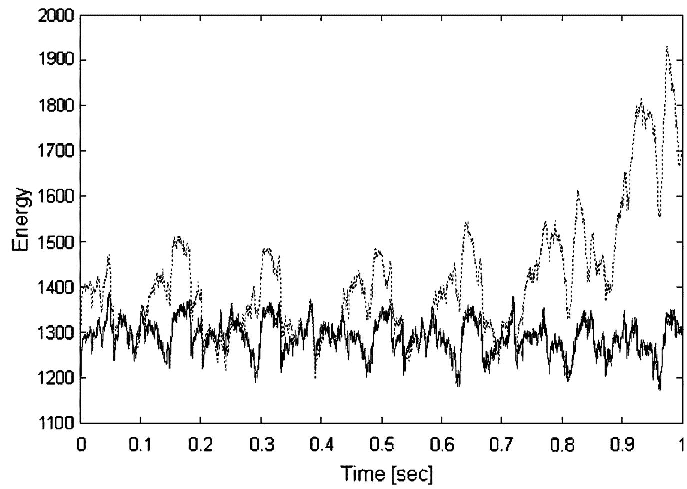



Fig. 3. Energies of uncontrolled translating beams tensioned as $T_0 = 10$ : $v = 1$ (solid). $v(t) = 1 + 0.5 \sin t$ (dashed). $v(t) = 1 + 0.5 \sin 40t$ (dotted).

*Remark 3:* Consider the uncontrolled span with the domain of $l < x < l_T$. In this case, the actuator position of $x = l$ becomes the left boundary of the span. Since the right boundary at $x = l_T$ is fixed, and the time-varying condition at the left boundary is really bounded due to the control action, the mechanical energy of the uncontrolled span is uniformly ultimately bounded, which can be simply obtained by evaluating the time derivation of the mechanical energy of the uncontrolled span part. However, the uniform ultimate boundedness region cannot be made arbitrarily small despite the control action at $x = l$ since the proposed robust adaptive controller was designed only by considering the dynamics of the controlled span.

The boundary controller proposed in (13)–(15) can also be directly applied to the axially moving string system without any modifications for ensuring the vibration reduction, since the dynamic model of a translating string with an arbitrarily varying speed can be easily obtained by setting $E = 0$ in the beam model (4)–(6).

## IV. Simulations and Discussion

The effectiveness of the proposed control laws and the verification of the introduced theories are demonstrated by numerical simulations. As mentioned, the vibrations of the uncontrolled span can be treated as actuator disturbance, and so only the controlled span given as (4)–(6) is presented here. For numerical simulations, consider the dimensionless variables [9], [10]. Then, the parameters of the beam and actuator in (4)–(6) are given as $\rho A = 1$, $l = 1$, $m_c = 0.5$, $e = 0$, $EI = 1$, and $EA = 1$. The disturbance $d(t)$ from the uncontrolled span is unknown, but for simulation purposes, $d(t) = 7 \sin 10t + 5 \cos 5t$ is given. Let the initial conditions of the beam satisfying the boundary conditions in (5) be $w(x, 0) = 10^2 \cdot x^2 \cdot (0.5 \cdot l - x)^3 \cdot (l - x)^2$ and $w_t(x, 0) = 0$, and let the initial conditions of the proposed controller be zero.

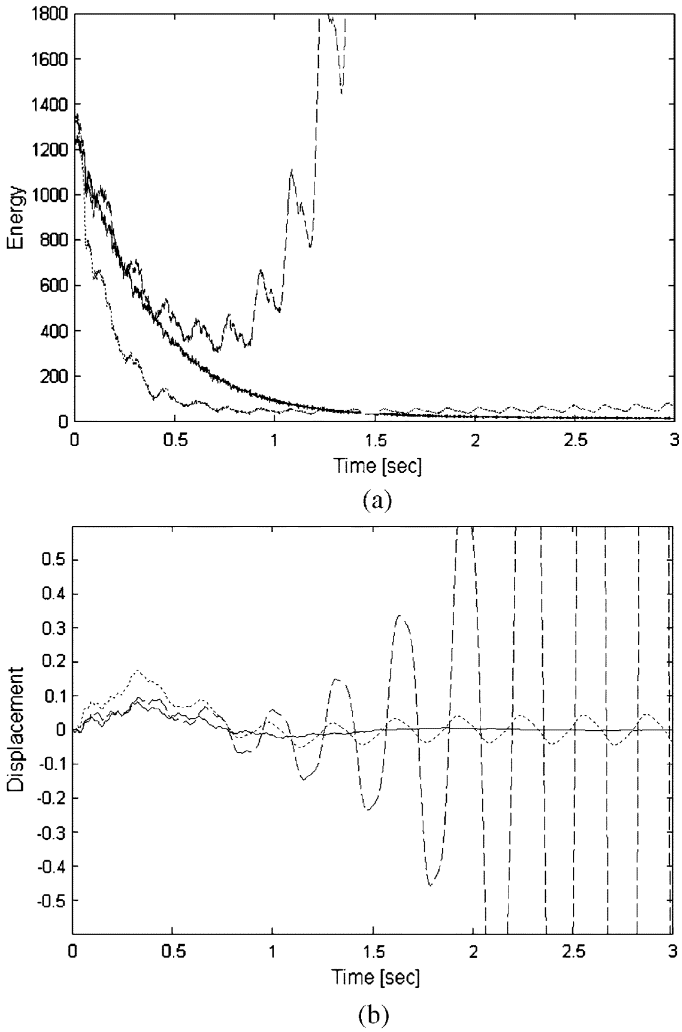



Fig. 4. Simulation results of closed-loop controlled system tensioned as $T_0 = 10$ with $v(t) = 1 + 0.5\sin t$ under $\beta = 0.3$ (solid); and with $v(t) = 1 + 0.5\sin 40t$ under $\beta = 0.3$ (dashed) and $\beta = 2.5$ (dotted). (a) Mechanical energy. (b) Boundary displacement.

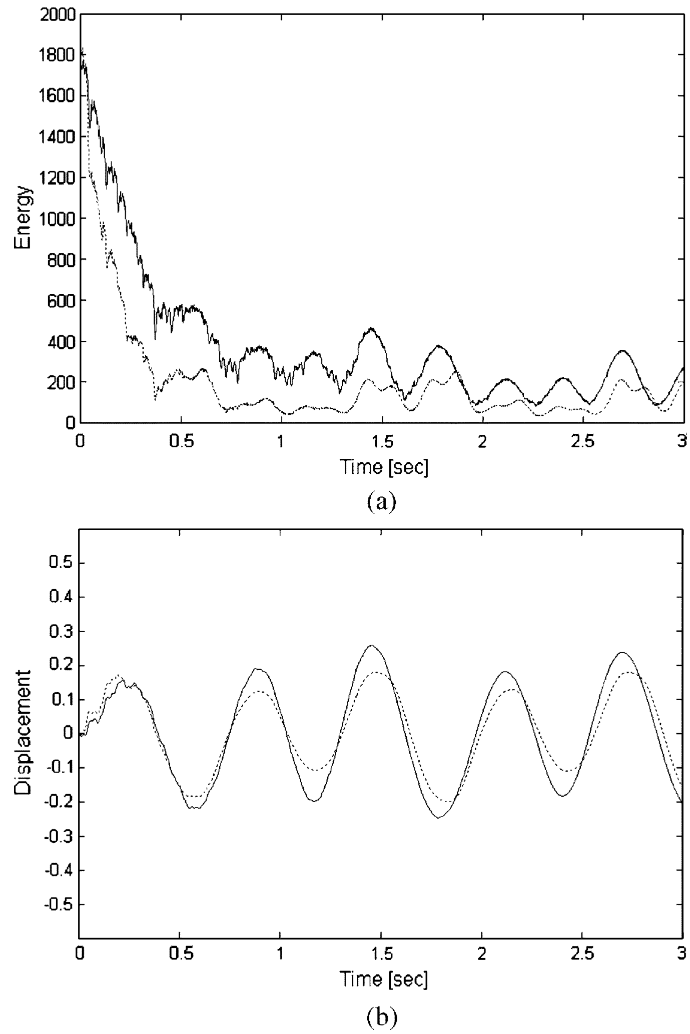



Fig. 5. Simulation results of closed-loop controlled systems tensioned as $T_0 = 100$ with $v(t) = 1 + 0.5\sin 5t$ under $\beta = 0.3$ (solid); and with $v(t) = 1 + 0.5\sin 40t$ under $\beta = 2.5$ (dotted). (a) Mechanical energy. (b) Boundary displacement.

### A. *Effect of Varying Speed $v(t)$ in Uncontrolled System*

Fig. 3 depicts the three types energy of the dimensionless beam in (4)–(6), disregarding the boundary disturbance and without both control force and damper; that is, $d = f_c = d_c = 0$ in (6), under $T_0 = 10$ and $v = 1$ (solid line), $v(t) = 1 + 0.5\sin t$ (dashed line), and $v(t) = 1 + 0.5\sin 40t$ (dotted line), respectively. As shown in Fig. 3, the energies of the beam systems with $v = 1$ and $v(t) = 1 + 0.5\sin t$ remain level and slowly decrease, which reason is that the material mass $\rho A$ introduces a stabilizing effect to the translating continua [8]. Note that the difference of vibration energies between $v = 1$ and $v(t) = 1 + 0.5\sin t$ is not so large, despite the varying condition. However, the mechanical energy of the beam traveling at the speed of $v(t) = 1 + 0.5\sin 40t$ diverges as time passes due to the higher variation rate, as analyzed in Section II. As mentioned in Remark 1, the passive damper alone is not adequate to suppress the vibrations. In [7], it was shown that the performance of the open-loop controlled system with a higher damping value is inferior to that with a lower damping value where the system might be unstable under the condition of a higher variation rate. Hence, in 2) following, the proposed feedback boundary controller proposed will be applied to the beam systems with the speed variations of $v(t) = 1 + 0.5\sin t$ and $v(t) = 1 + 0.5\sin 40t$.

### B. *Effect of Varying Speed $v(t)$ in Controlled Systems*

Fig. 4 shows the simulation results for the closed-loop controlled beam system having the control gain $\beta = 0.3$ in (13) about the two traveling speeds of $v(t) = 1 + 0.5\sin t$ (solid line) and $v(t) = 1 + 0.5\sin 40t$ (dashed line), respectively, under the assumption of $d = f_d = 0$, where the mechanical energy and the boundary displacement at $x = l$ are depicted in (a) and (b), respectively. As analyzed in Remark 1, in the case of the closed-loop controlled system with $v(t) = 1 + 0.5\sin t$, which satisfies the conditions in Theorem 1, the vibrational energy is exponentially reduced. However, in the case of the beam system with a much faster variation $v(t) = 1 + 0.5\sin 40t$, the vibration energy diverges despite the boundary controller, since the faster varying property forbids the system conditions to satisfy those in Theorem 1.

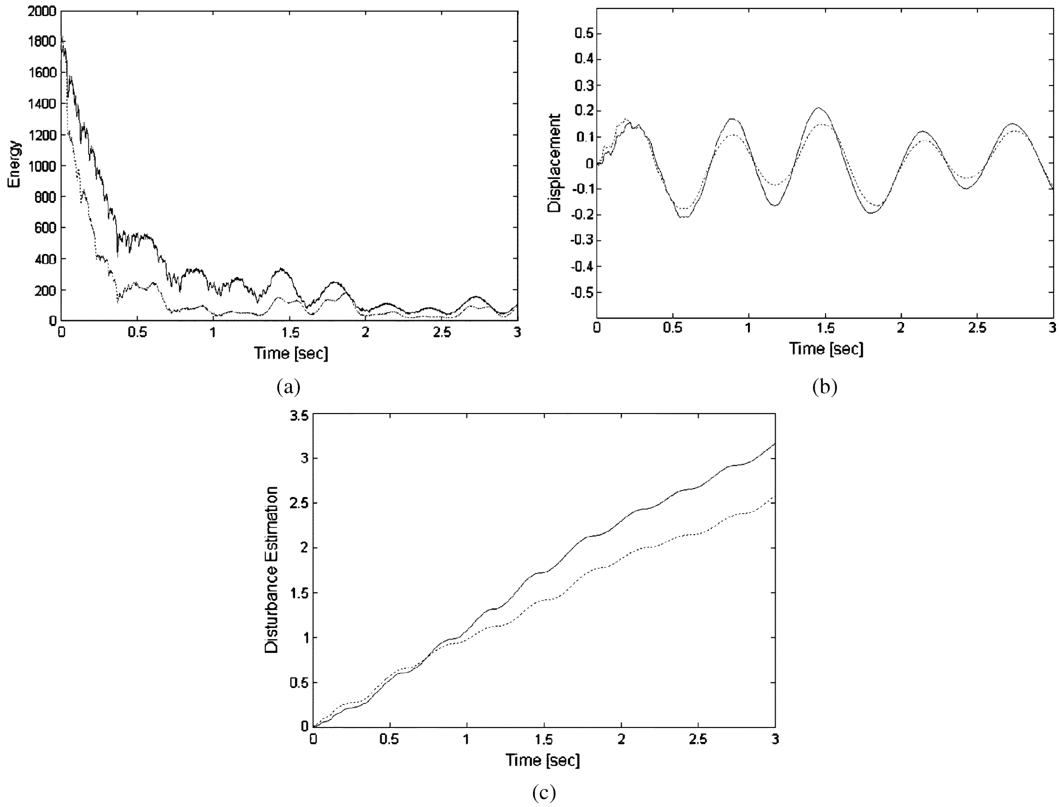



Fig. 6. Simulation results of closed-loop controlled systems tensioned as $T_0 = 100$ with $v(t) = 1 + 0.5 \sin 5t$ under $\beta = 0.3$ and $\gamma_d = 1$ (solid); and with $v(t) = 1 + 0.5 \sin 40t$ under $\beta = 2.5$ and $\gamma_d = 1$ (dotted). (a) Mechanical energy. (b) Boundary displacement. (c) Disturbance estimation.

The unpredictably quickly varying patterns of the moving speed can be somewhat overcome by selecting a higher value of control gain $\beta$, which was observed in [7]. Following this observation, the beam system with the traveling speed of $v(t) = 1 + 0.5 \sin 40t$ is simulated again by employing the active boundary controller setting of $\beta = 2.5$ instead of $\beta = 0.3$, and which result is depicted as the dotted line in Fig. 4. Compared with the vibration energy of the closed-loop controlled system with $\beta = 0.3$, the performance of the controlled system with the higher control gain, $\beta = 2.5$, is significantly improved, and the divergence no longer takes place, despite the fast variation. However, Fig. 4 also indicates that the vibrations of the fast varying system controlled by higher control gain $\beta$ still remain at a level, that they do not converge to zero, and that the boundary displacement is also continuously disturbed despite the disregarding of the boundary disturbance.

### C. Effect of Disturbance $d(t)$ in Controlled Systems

Now consider the total span containing the uncontrolled span shown in Fig. 2; that is, the unknown disturbance $d(t)$ is surely considered in the controlled system. Also, in actual situations, such the worst phenomena as the dotted line in Fig. 3 might be unreasonable, and then the initial tension applied to the beam is given as $T_0 = 100$ instead of $T_0 = 10$ in the remainder. Note that the closed-loop system with $T_0 = 100$ can be easily expected the exponential stability under the same control conditions as those in 2) from Remark 1. To investigate whether the same simulation results as those in Fig. 4 can get under the boundary disturbance, the time-varying beam with $v(t) = 1 + 0.5 \sin 5t$ controlled by $\beta = 0.3$ (solid line) and with $v(t) = 1 + 0.5 \sin 40t$ controlled by $\beta = 2.5$ (dotted line), respectively, was simulated without employing the robust control term $f_d$, that is, $f_d = 0$ in (13), and the results are shown in Fig. 5. From Fig. 5, it is clearly seen that the vibration energy of the closed-loop system with $v(t) = 1 + 0.5 \sin 5t$ controlled by $\beta = 0.3$ no longer converges to zero and remains level, not considering the controlled system with faster variation, $v(t) = 1 + 0.5 \sin 40t$. Comparing Fig. 5 with Fig. 4(b), it is also noted that the magnitude of the boundary displacement was additionally increased. Such deterioration of the control performance is definitely due to the boundary disturbance $d(t)$ from the uncontrolled span. Hence, as analyzed in Section III, when the effect of the vibrations from the adjacent span is felt, the boundary control law $f_c$ itself cannot guarantee the expected performance in actual situations. To overcome the unknown undesired vibrational effect from the uncontrolled span, the robust adaptation control terms proposed in (14), (15) should be properly added into the boundary control law $f_c$ in (13).

Now, the robust adaptive boundary controller in (13)–(15) is employed under the same controlled conditions as those in Fig. 5. The results are presented in Figs. 6 and 7, where the robust control gains are set to $\gamma_d = 1$ and $\gamma_d = 10$, respectively, and to $\delta_d = 10^{-3}$ and $\varepsilon_d = 10^{-3}$ in common. In Fig. 7, the robust adaptive controller is also applied to the faster varying system with $\beta = 2.5$ in Fig. 4, and the results are depicted as dashed line. Graphs (a), (b), and (c) in Fig. 6 and 7 denote the mechanical energy, the boundary displacement, and the disturbance estimation, respectively. Note that the convergence of the estimated parameter to the exact value is not essential in this control scheme.

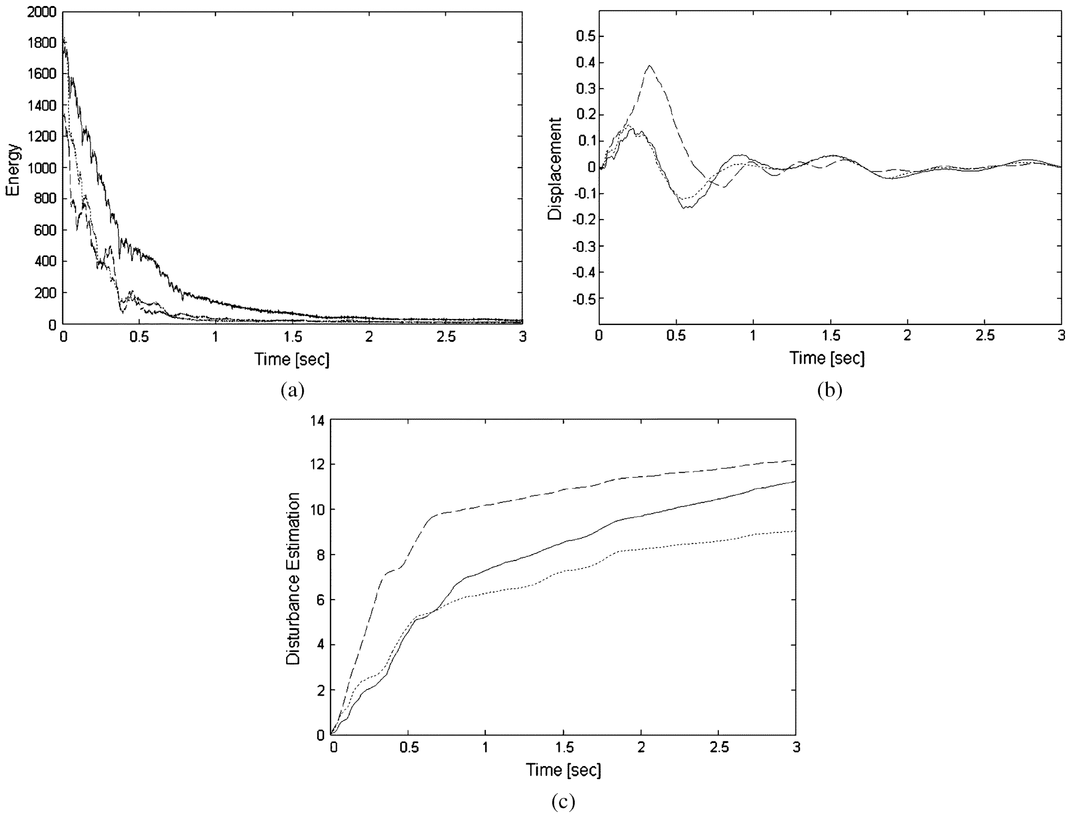



Fig. 7. Simulation results of closed-loop controlled systems tensioned as $T_0 = 100$ with $v(t) = 1 + 0.5 \sin 5t$ under $\beta = 0.3$ and $\gamma_d = 10$ (solid); and with $v(t) = 1 + 0.5 \sin 40t$ under $\beta = 2.5$ and $\gamma_d = 10$ (dotted); and tensioned as $T_0 = 10$ with $v(t) = 1 + 0.5 \sin 40t$ under $\beta = 2.5$ and $\gamma_d = 10$ (dotted). (a) Mechanical energy. (b) Boundary disturbance. (c) Disturbance estimation.

From Figs. 6(a) and 7(a), it is observed that, by adding the robust term $f_d$ and the adaptive estimation $\hat{\mu}_d$ into the boundary control law $f_c$, the effect of the undesired disturbance can be considerably suppressed, and the situation becomes outstanding when setting the higher control gain $\gamma_d$, as explained in Theorem 1. Comparing Fig. 7(b) with Fig. 4(b), where not even the boundary disturbance is contained, it is also noted that the boundary displacement of the fast varying system with $T_0 = 10$ controlled by the robust adaptive controller is no longer disturbed and converges near to zero in a very stable manner as time passes. This result explains that the robustness property of the proposed controller is effective not only against boundary disturbances but also against the uncertainly varying patterns of the moving speed.

## D. Effect of Control Gain $\beta$ in Closed-Loop System

From the discussion of 2), it was seen that the boundary controller can attain a certain robustness against the time variations of moving speed if a higher control gain $\beta$ is selected. Hence, now it is investigated whether the control law $f_c$ in (13) can eliminate the effect of undesired boundary disturbances only by using the higher control gain $\beta$ without employing the robust control term $f_d$, (i.e., $f_d = 0$). For this, $\beta = 5$ is given to the boundary control law $f_c$, and a closed-loop system, under the same parameter and disturbance conditions as those in Fig. 5, is considered (the results are depicted in Fig. 8). Comparing Fig. 8(a) with Figs. 5(a)–7(a), it is seen that the vibration energy of beam still remains at a level not converging near to zero despite the higher control gain $\beta$, the initial converging rate of vibration energy increases though. Also, the difference of vibration energies between $v(t) = 1 + 0.5 \sin 5t$ and $v(t) = 1 + 0.5 \sin 40t$ is not so large, despite the varying condition. However, the main result to be noted is shown in Fig. 8(b); that is, that the magnitude of the boundary displacements is still maintained at a high value and the disturbance are almost the same, despite the time-varying condition. This means that the effect of the boundary disturbance is still manifest, despite the higher control gain $\beta$.

Thus, it is concluded that the higher control gain $\beta$ itself might be insufficient to overcome the effect of the boundary disturbances without the cooperation of the robust control term $f_d$, even though the higher gain surely imparts a robustness to the boundary controller $f_c$. However, the robustness property from the higher control gain $\beta$ should not be underestimated in the control problem for continua systems, least to all, for slowly-varying or stationary continua systems [7].

## E. Instability of Uncontrolled Side Measurement

In Section III, it was mentioned that, if the boundary slope as the control input signal is measured on the uncontrolled span side of the actuator, the closed-loop system can be unstable. Now suppose a downstream moving beam, that is, $v(t) < 0$ in (4)–(6), with disregarding the boundary disturbance and robust control term ($d = f_d = 0$ in (6)). Fig. 9 describes the simulation results for the beam traveling at the downstream speed of $v(t) = -1 - 0.5 \sin 5t$, in order to compare the open-loop controlled system, operated by only the passive damper setting $d_c = 50$ (dotted line), with the closed-loop systems having the

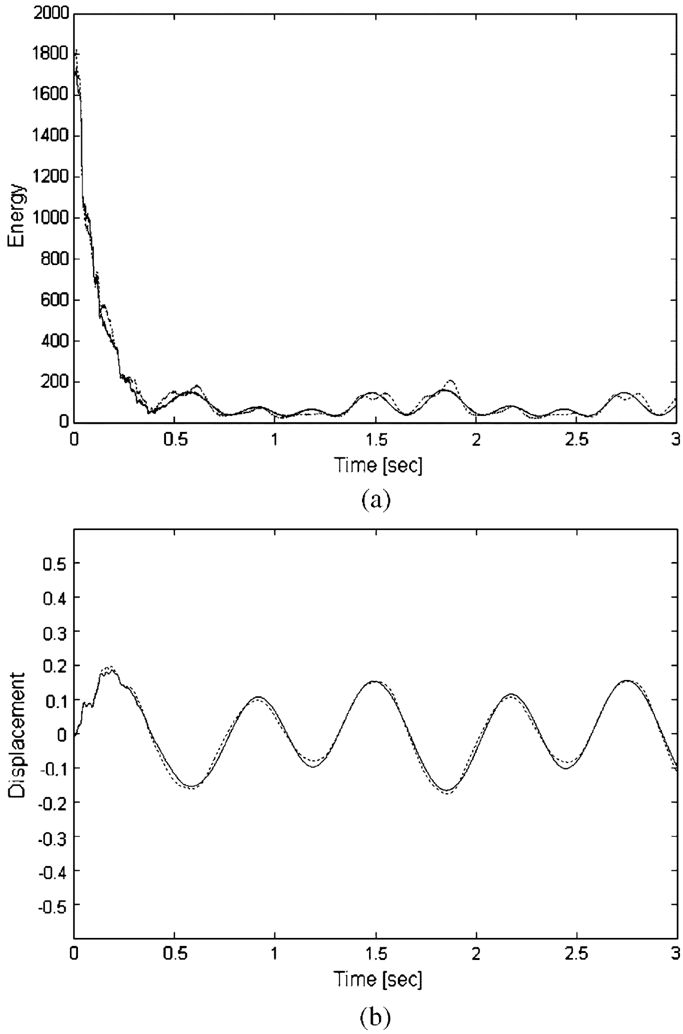



Fig. 8. Simulation results of closed-loop controlled systems tensioned as $T_0 = 100$ with $v(t) = 1 + 0.5 \sin 5t$ under $\beta = 5$ (solid); and with $v(t) = 1 + 0.5 \sin 40t$ under $\beta = 5$ (dotted). (a) Mechanical energy. (b) Boundary disturbance.

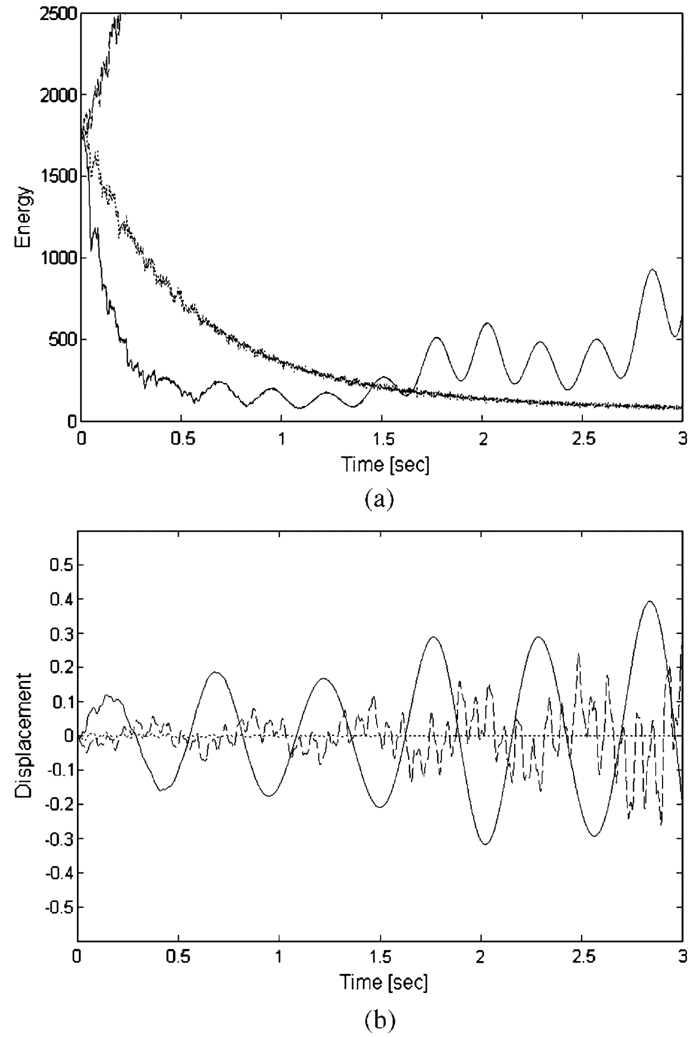



Fig. 9. Simulation results of open-loop and closed-loop controlled systems with $T_0 = 100$ and $v(t) = -1 - 0.5 \sin 5t$; $d_c = 50$ (dotted line), $\beta = 0.3$ (dashed line), and $\beta = 3.5$ (solid line). (a) Mechanical energy. (b) Boundary displacement.

control gain $\beta = 0.3$ (dashed line) and $\beta = 5$ (solid line), respectively. As shown in Fig. 9(a), the vibration energies of the downstream moving beam with the proposed boundary controller diverge as time passes even with the higher control value given by $\beta = 5$, whereas the energy of the open-loop controlled system is stabilized in a stable manner.

The boundary displacements of the closed-loop systems are also gradationally increasing, as shown in Fig. 9(b). Hence, on the basis of the robustness of $f_d$ against boundary disturbances, the robust control term $f_d$ in (14) is added again with the same control gains as those used in Fig. 7 to compensate the unstable boundary conditions. The simulation results are depicted in Fig. 10, from which it is seen that the unstable disturbance displacements at the boundary are significantly reduced [Fig. 10(b)] Also, in the case of the closed-loop system having $\beta = 5$, the vibrational energy has been stabilized more quickly than that of the open-loop system, and converges to zero as well without any divergence [Fig. 10(a)]. However, it is also noted that the closed-loop system controlled by $\beta = 0.3$ is still unstable, despite the robust controller. Hence, when applying the proposed boundary controller to downstream moving continua, the control gain $\beta$ should be carefully selected as a suitable value, and, needless to say, the robust control term $f_d$ should be added.

Figs. 9 and 10 inform that the control actuator in Fig. 2 should have the input signals measured on the controlled span side of the actuator, not on the uncontrolled span side, because otherwise the closed-loop system might be unstable despite the boundary control action. Hence, in the case of upstream moving continua controlled by a right boundary actuator, if possible, it is better to avoid a control scheme designed by using an input signal measured on the uncontrolled span side of the actuator.

From the simulation results and discussions in 1)–5), it is finally summarized that, under the robust adaptive boundary control law proposed in (13)–(15), the vibrational energy of translating continua systems with an arbitrarily varying speed can be stabilized and effectively dissipated by setting appropriate control gains.

## V. Conclusion

In this brief, a robust adaptive boundary control scheme for axially moving continua with an arbitrarily varying speed has

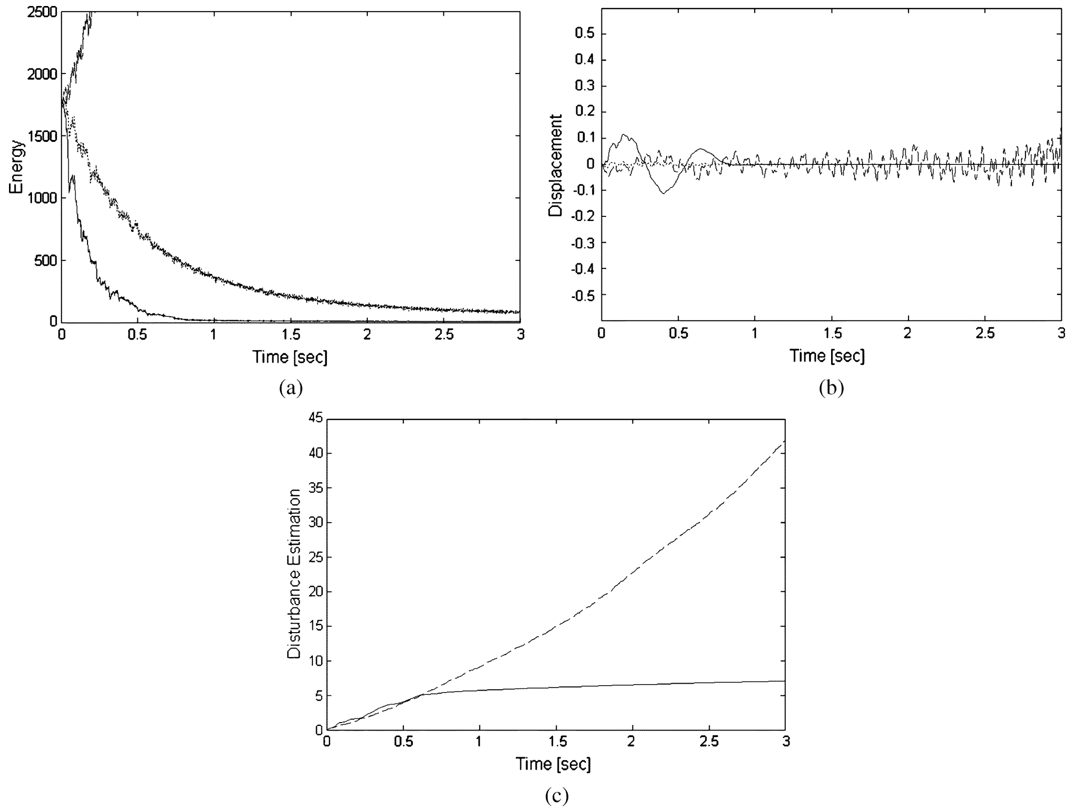



Fig. 10. Simulation results of open-loop and closed-loop controlled systems with $T_0 = 100$ and $v(t) = -1 - 0.5\sin 5t$; $d_c = 50$ (dotted line), $\beta = 0.3$ and $\gamma_d = 10$ (dashed line), and $\beta = 3.5$ and $\gamma_d = 10$ (solid line). (a) Mechanical energy. (b) Boundary displacement. (c) Disturbance estimation.

been proposed. For axially translating continua, three things are essential to design an effective vibration controller: The span of interest in the continua system is securely connected to the adjacent span and, hence, the effect from the motion of the adjacent span should be properly treated for ensuring the vibration reduction of the span of interest; due to the continuity property of the materials, the slope term on the controlled span side of the actuator should be included in the Lyapunov energy functional, especially, in the energy term of the actuator; and since an effective control law can be derived by using the time rate of change of the Lyapunov functional, the slope rate at the right boundary is then required as an input signal to the controller. By properly handling this signal, the vibrations of translating as well as stationary continua can be more effectively suppressed.

## REFERENCES

[1] D. Chen, "Adaptive control of hot-dip galvanizing," *Automatica*, vol. 31, no. 5, pp. 715–733, 1995.

[2] K. J. Yang, K. S. Hong, and F. Matsuno, "Robust boundary control of an axially moving string by using a PR transfer function," *IEEE Trans. Autom. Control* , to be published.

[3] K. J. Yang, K. S. Hong, and F. Matsuno, "Robust adaptive boundary control of an axially moving string under a spatiotemporally varying tension," *J. Sound Vib.*, vol. 273, pp. 1007–1029, 2004.

[4] S. Y. Lee and C. D. Mote, "Vibration control of an axially moving string by boundary control," *ASME J. Dyn. Syst. Meas. Control*, vol. 118, pp. 66–74, 1996.

[5] R. F. Fung and C. C. Tseng, "Boundary control of an axially moving string via Lyapunov method," *ASME J. Dyn. Syst. Meas. Control*, vol. 121, no. 1, pp. 105–110, 1999.

[6] Y. Li, D. Aron, and C. D. Rahn, "Adaptive vibration isolation for axially moving strings: theory and experiment," *Automatica*, vol. 38, no. 3, pp. 379–390, 2002.

[7] K. J. Yang, K. S. Hong, and F. Matsuno, "Boundary control of a translating tensioned beam with varying speed," *IEEE/ASME Trans. Mechatronics* , Oct. 2005, to be published.

[8] W. D. Zhu and J. Ni, "Energetics and stability of translating media with an arbitrarily varying length," *ASME J. Vib. Acoust.*, vol. 122, pp. 295–304, 2000.

[9] J. A. Wickert, "Non-linear vibration of a traveling tensioned beam," *Int. J. Nonlinear Mech.*, vol. 27, no. 3, pp. 503–517, 1992.

[10] S. Y. Lee and C. D. Mote, "Wave characteristics and vibration control of translating beams by optimal boundary damping," *ASME J. Dyn. Syst. Meas. Control*, vol. 121, pp. 18–25, 1999.

[11] Y. Li and C. D. Rahn, "Adaptive vibration isolation for axially moving beams," *IEEE/ASME Trans. Mechatronics*, vol. 5, pp. 419–428, 2000.

[12] K. J. Yang, K. S. Hong, and F. Matsuno, "Robust adaptive control of a cantilevered flexible structure with spatiotemporally varying coefficients and bounded disturbance," *JSME Int. J. Ser. C*, vol. 47, pp. 812–822, 2004.

[13] J. Bentsmann and Y. V. Orlov, "Reduced spatial order model reference adaptive control of spatially varying distributed parameter systems of parabolic and hyperbolic types," *Int. J. Adapt. Control Signal Process.*, vol. 15, no. 6, pp. 479–696, 2001.

[14] M. P. Fard and S. I. Sagatun, "Exponential stabilization of a transversely vibrating beam via boundary control," *J. Sound Vib.*, vol. 240, no. 4, pp. 613–622, 2001.

[15] K. J. Yang, K. S. Hong, and F. Matsuno, "The rate of change of an anergy functional for axially moving continua," in *IFAC 16th World Congress*, Praha, Czech Republic, Jul. 4–8, 2005.

[16] W. D. Zhu, "Control volume and system formulations for translating media and stationary media with moving boundary," *J. Sound Vib.*, vol. 254, no. 1, pp. 189–201, 2002.

[17] H. Benaroya and Y. Wei, "Hamilton's principle for external viscous fluid-structure interaction," *J. Sound Vib.*, vol. 238, no. 1, pp. 113–145, 2000.

[18] J. A. Wichert and C. D. Mote Jr., "On the energetics of axially moving continua," *J. Acoust. Soc. Amer.*, vol. 85, no. 3, pp. 1365–1368, 1989.

[19] M. J. Corless and G. Leitmann, "Continuous state feedback guaranteeing uniform boundedness for uncertain dynamic systems," *IEEE Trans. Autom. Control*, vol. AC–26, no. 5, pp. 1139–1144, Oct. 1981.